

Q If $ax^2+bx+c=0$ & $bx^2+cx+a=0$ have a Com. Root then $\frac{a^3+b^3+c^3}{abc}=?$

$$\begin{array}{ccc} a & b & c \\ b & c & a \end{array}$$

$$(ac-b^2)(ab-c^2) = (a^2-bc)^2$$

$$a^2bc - ac^3 - ab^3 + b^2c^2 = a^4 + b^2c^2 - 2a^2bc$$

$$abc - c^3 - b^3 = a^3 - 2abc$$

$$3abc = a^3 + b^3 + c^3$$

$$\frac{a^3+b^3+c^3}{abc} = 3$$

$$\underbrace{a^3+b^3+c^3-3abc=0} \begin{cases} a+b+c=0 \\ a^2+b^2+c^2-ab-bc-ca=0 \end{cases}$$

Q If $x^2+bx+c=0$ & $bx^2+cx+1=0$ have

1 Com. Root then P.T. $b+c+1=0$ or

$$\underline{b^2+c^2+1 = bc+cb+c}$$

$$\begin{array}{ccc} 1 & b & c \\ b & c & 1 \end{array}$$

$$(c-b^2)(b-c^2) = (1-bc)^2$$

$$bc - c^3 - b^3 + b^2c^2 = 1 + b^2c^2 - 2bc$$

$$b^3+c^3+1^3-3bc=0 \Rightarrow \boxed{b+c+1=0}$$

$$b^2+c^2+1^2-bc-c\cdot 1-b\cdot 1=0$$

$$\boxed{b^2+c^2+1 = bc+cb+c}$$

Q $x^2 + px + q = 0$ & $x^2 + qx + p = 0$ have

1 Com. Root then S.T. $\boxed{1+p+q=0}$

& also show that Uncommon Roots

are Roots of $x^2 + x + pq = 0$.

$$\left. \begin{array}{l} x^2 + px + q = 0 \\ x^2 + qx + p = 0 \end{array} \right\} \begin{array}{l} \alpha \cdot \beta = q \Rightarrow 1 \cdot \beta = q \Rightarrow \boxed{\beta = q} \\ \alpha \cdot \gamma = p \Rightarrow 1 \cdot \gamma = p \Rightarrow \boxed{\gamma = p} \end{array}$$

$\therefore$ Uncommon Roots β & γ

$$x^2 - (\beta + \gamma)x + \beta\gamma = 0$$

$$x^2 - (p + q)x + pq = 0$$

$$x^2 - (-1)x + pq = 0$$

$$x^2 + x + pq = 0$$

$$\begin{array}{r} 1^2 + p \cdot 1 + q = 0 \\ \boxed{p + q = -1} \\ \text{J.I.P.} \\ \hline x^2 + px + q = 0 \\ x^2 + qx + p = 0 \\ \hline (p - q)x + q - p = 0 \\ (p - q)x = p - q \\ \boxed{x = 1} \end{array}$$

Q. If eqn $x^3 + x^2 - 4x - 4 = 0$ & $x^2 + px + 2p = 0$

$p \in \mathbb{R}$ have 2 com. Roots then $p = ?$

$$x^3 + x^2 - 4x - 4 = 0$$

$$x^2(x+1) - 4(x+1) = 0$$

$$(x+1)(x^2 - 4) = 0$$

$$(x+1)(x-2)(x+2) = 0$$

$$(\underline{x^2 - x - 2})(x+2) = 0$$

$$\begin{array}{l} (x^2 + 3x + 2)(x-2) \\ (x^2 + px + 2p) \end{array}$$

$$\underline{x^2 + px + 2p = 0} \quad \left\{ \begin{array}{l} \text{compare} \end{array} \right.$$

$$p = -1$$

Q. If monic quad trinomial $P(x)$ is such

that $P(x) = 0$ & $\underline{P(P(P(x)))} = 0$ have a

common Root then $P(0) \cdot P(1)$

< 0

> 0

$= 0$

NOT

$$P(x) = \underline{x^2 + ax + b}$$

$$P(0) = b \quad \& \quad a + b + 1 = P(1)$$

$$P(x) = 0 \quad P(\underline{P(P(x))}) = 0$$

$$\downarrow$$

$$P(\underline{P(0)}) = 0$$

$$P(b) = 0$$

$$b^2 + ab + b = 0$$

$$\underline{(b)}(b+a+1) = 0$$

$$P(0) \cdot P(1) = 0$$

Q Supp. a Cubic Poly $f(x) = x^3 + px^2 + qx + 72$ is div. by $x^2 + ax + b$ & $x^2 + bx + a$. Find Sum.

of Sq^r of Roots of Cubic Poly.

$$\begin{aligned} x^2 + ax + b = 0 & \quad \alpha, \beta \\ x^2 + bx + a = 0 & \quad \gamma \end{aligned} \quad \left. \begin{aligned} 1 \cdot \beta = b \Rightarrow \boxed{\beta = b} \\ 1 \cdot \gamma = a \Rightarrow \boxed{\gamma = a} \end{aligned} \right\}$$

$$\begin{aligned} & \boxed{x^2 + ax + b = 0} \\ & \boxed{x^2 + bx + a = 0} \\ & \underline{-(a-b)} \\ & \alpha(a-b) = (a-b) \\ & \alpha \neq 1 \Rightarrow \boxed{\alpha = 1} \\ & 1^2 + a + b = 0 \\ & 1 + a + b = 0 \\ & 1 + \gamma + \beta = 0 \Rightarrow \boxed{\beta + \gamma = -1} \end{aligned}$$

$$\begin{aligned} \beta + \gamma = -1 \\ \beta \gamma = -72 \end{aligned} \quad \left. \begin{aligned} \beta = -9, \gamma = 8 \end{aligned} \right\}$$

Sum of Sq^r of Roots.

$$\begin{aligned} 1^2 + \beta^2 + \gamma^2 &= 1^2 + (-9)^2 + 8^2 \\ &= 81 + 1 + 64 \\ &= 146 \end{aligned}$$

$$(x-1)(x-2)(x-3) \text{ div by } (x-1)(x-2)$$

$$(x-1)(x-2)(x-3) \text{ div by } (x-1)(x-3)$$

1 Cubic Poly 2 different Quad Se. div. ho

to Undo Quad me 1 Root
com hoga.

$$\begin{array}{l|l|l} \alpha + \beta + \gamma = -p & \alpha\beta + \beta\gamma + \gamma\alpha = q & \alpha\beta\gamma = -72 \\ 1 + a + b = -p & \underline{1} + \underline{\beta\gamma} + \underline{\gamma} = q & 1 \cdot \beta\gamma = -72 \\ 0 = -p & -1 - 72 = q & \boxed{\beta\gamma = -72} \\ p = 0 & q = -73 & \end{array}$$

Quad Poly of 2 Variable [Coordinate]

$$1) f(x, y) = \underline{ax^2 + by^2 + 2hxy + 2gx + 2fy + c}$$

is Quad Poly of 2 Variable

2) This Quad Poly can be reduced in 2 linear fxn. if Condⁿ $\Delta = 0$ is Satisfied.

$$3) \Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Q Find value of m for which Exp.

$y^2 + 2xy + 2x + my - 3 = 0$ is Resolved in 2 linear factors.

$$+ y^2 + 2xy + 2x + my - 3 = 0$$

$$\underline{ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0}$$

$$\downarrow$$

$$a=0, b=1, h=1, g=1, 2f=m, c=-3$$

$$f = \frac{m}{2}$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 0 \cdot 1 \cdot (-3) + 2 \cdot \frac{m}{2} \cdot 1 \cdot 1 - 0 \cdot \left(\frac{m}{2}\right)^2 - 1 \cdot 1^2 - (-3)(1)^2$$

$$\Delta = m - 1 + 3 \Rightarrow \Delta = m + 2$$

if given Quad Poly is reducible then $\Delta = 0$
 $m + 2 = 0 \Rightarrow \boxed{m = -2}$

Q. Find Linear factors of

$$x^2 + 3xy + 2y^2 - 2x - 3y - 35 = 0$$

→ Quad Poly in 2 var.

if it is Reducible into linear factors

$$\Delta = 0$$

make Quad Eqn in y

$$2y^2 - 3y - 3xy + x^2 - 2x - 35 = 0$$

$$2y^2 - y(3+3x) + (x^2 - 2x - 35) = 0$$

$$y = \frac{-(3+3x) \pm \sqrt{(3+3x)^2 - 4 \times 2 \times (x^2 - 2x - 35)}}{2 \cdot 2}$$

$$= \frac{-(3+3x) \pm \sqrt{9x^2 + 9 + 18x - 8x^2 + 16x + 280}}{4}$$

$$y = \frac{(3+3x) \pm \sqrt{x^2 + 34x + 289}}{4}$$

$$= \frac{(3+3x) \pm \sqrt{(x+17)^2}}{4}$$

$$y = \frac{3+3x+x+17}{4}, \frac{3+3x-(x+17)}{4}$$

$$y = x+5, \quad y = \frac{x}{2} - \frac{7}{2}$$

Q Least value of Expression

$$x^2 + 2x + 2y^2 + 4y + 7 \text{ ?}$$

$$\text{A. } x^2 + 2x + y^2 + (y^2 + 4y + 4) + 3$$

$$= (x+1)^2 + (y+2)^2 + 3$$

$$\geq 0 \quad \geq 0 \quad \downarrow$$

$$\geq 0 + 3$$

$$\text{Exp.} \geq 3$$

$$\text{Least value} = 3$$

Q Condⁿ for which

$a^2x^4 + bx^3 + cx^2 + dx + f^2$ is a Perfect sq.

$$\text{in } 2a^2c = a^3f \quad 4a^2c - b^2 = 8a^3f$$

$$4a^3c = 8a^3f \quad \text{None}$$

$$\boxed{a^2x^4} + \underline{bx^3 + cx^2 + dx} + \boxed{f^2}$$

$$= (ax^2 + \boxed{mx} + f)^2$$

↓ assume
open & compare

Transformation of Roots

$$ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

New Roots.		New Eqn.
$\frac{1}{\alpha}, \frac{1}{\beta}$	$\rightarrow y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$	$a\left(\frac{1}{y}\right)^2 + b\left(\frac{1}{y}\right) + c = 0$
$\alpha + k, \beta + k$	$\rightarrow y = \alpha + k$ $\alpha = y - k$	$a(y - k)^2 + b(y - k) + c = 0$
$k\alpha, k\beta$	$\rightarrow y = kx$ $x = \frac{y}{k}$	$a\left(\frac{y}{k}\right)^2 + b\left(\frac{y}{k}\right) + c = 0$
α^2, β^2	$\rightarrow y = x^2$ $x = \pm\sqrt{y}$	$a(\pm\sqrt{y})^2 + b(\pm\sqrt{y}) + c = 0$

Q $2x^2 + 2x - 1 = 0$ has 2 Roots α & β

find Eqn whose Roots are $\boxed{-\frac{2}{\alpha}}$ & $\boxed{\frac{2}{\beta}}$?
New Roots

$$-\frac{2}{\alpha} \Rightarrow y \Rightarrow \boxed{x} = -\frac{2}{y}$$

$$2\left(-\frac{2}{y}\right)^2 + \left(-\frac{2}{y}\right) - 1 = 0$$

$$\frac{8}{y^2} - \frac{2}{y} - 1 = 0$$

$$8 - 2y - y^2 = 0$$

$$y^2 + 2y - 8 = 0$$

$$\boxed{x^2 + 2x - 8 = 0}$$

Q $x^2 + 5x + 4 = 0$ has 2 Roots α & β .

find Eqⁿ whose Roots are $\frac{\alpha+2}{3}, \frac{\beta+2}{3}$

$$y = \frac{\alpha+2}{3} \Rightarrow 3y = \alpha+2 \Rightarrow \alpha = (3y-2)$$

$$(3y-2)^2 + 5(3y-2) + 4 = 0$$

$$9y^2 - 12y + 4 + 15y - 10 + 4 = 0$$

$$9y^2 + 3y - 2 = 0$$

$$9x^2 + 3x - 2 = 0$$

Q $3x^2 - 5x + 2 = 0$ has Root α, β find Eqⁿ whose Roots are $\alpha^2 - 2, \beta^2 - 2$

$$y = \alpha^2 - 2 \Rightarrow \alpha^2 = y + 2 \Rightarrow \alpha = \sqrt{y+2}$$

$$3(\sqrt{y+2})^2 - 5\sqrt{y+2} + 2 = 0$$

$$3y + 6 + 2 = 5\sqrt{y+2}$$

$$(3y+8)^2 = (5\sqrt{y+2})^2$$

$$9y^2 + 64 + 48y = 25y + 50$$

$$9y^2 + 23y + 14 = 0$$

$$9x^2 + 23x + 14 = 0$$

Q $5x^2 - 7x + 3 = 0 \xrightarrow{\alpha, \beta} \text{New Eqn.}$

whose Roots $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

$$y = \frac{\alpha-1}{\alpha+1} \Rightarrow \alpha + y = \alpha - 1$$

$$\Rightarrow \alpha(y-1) = -1-y$$

$$\Rightarrow \alpha = \frac{1+y}{1-y}$$

$$5\left(\frac{1+y}{1-y}\right)^2 - 7\left(\frac{1+y}{1-y}\right) + 3 = 0 \text{ Solve}$$

LOCATION OF ROOTS

LOR

Let $ax^2 + bx + c = 0 \xrightarrow{\alpha, \beta}$

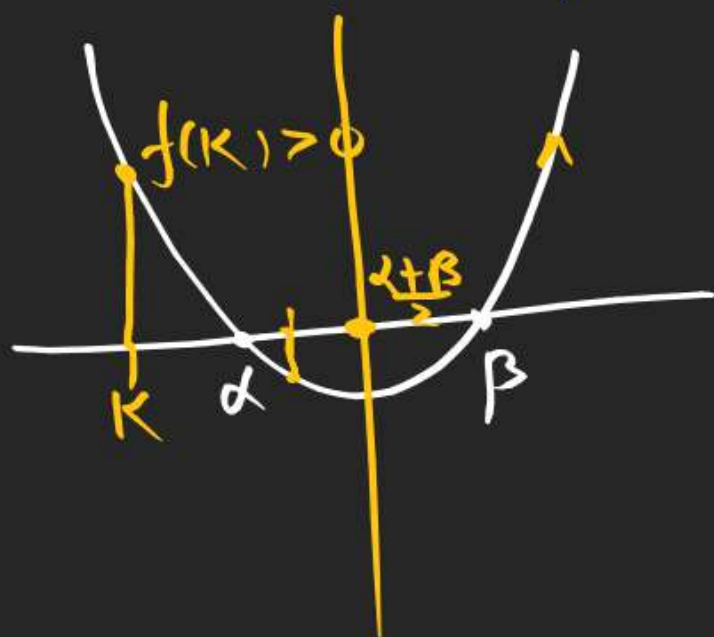
$$1 \cdot x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Let $f(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$



Profile 1 If Both Roots of $ax^2+bx+c=0$
are greater than some constant k .

$$f(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$$



$$\textcircled{1} D \geq 0$$

$$\textcircled{2} \alpha + \beta > k \Rightarrow -\frac{b}{2a} > k$$

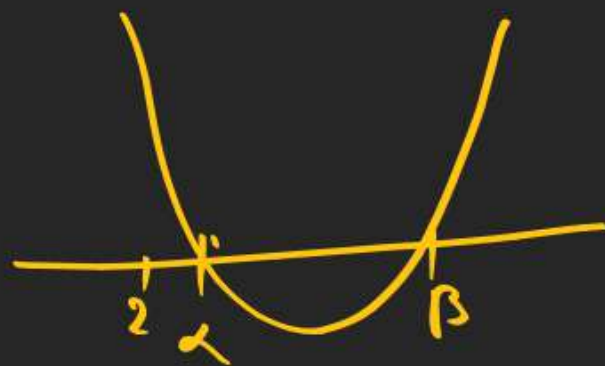
$$\textcircled{3} f(k) > 0$$

We will satisfy
above 3 condⁿ

Q Find k for which Roots of Eqn.

$x^2 - kx + k = 0$ are gr. than $\boxed{2}$.

(Corresponds) $f(x) = x^2 - kx + k$
fxn



① $D \geq 0$

$$(-k)^2 - 4 \cdot k \geq 0$$

$$k^2 - 4k \geq 0$$

$$(k)(k-4) \geq 0$$

$$\underline{k \leq 0 \cup k \geq 4}$$

② $-\frac{b}{2a} > 2$

$$\frac{-(-k)}{2 \cdot 1} > 2$$

$$\boxed{k > 4}$$

③ $f(2) > 0$

$$2^2 - 2k + k > 0$$

$$\boxed{k < 4}$$

$$\underline{k = \phi}$$